Model-Independent Algorithms for Time-Optimal Control of Chemical Processes

Yeong-Iuan Lin, John N. Beard, and Stephen S. Melsheimer
Dept. of Chemical Engineering, Clemson University, Clemson, SC 29634

The new time-optimal control algorithms presented do not require a-priori knowledge of the process dynamics. They are applicable to systems that can be described by a second-order (or first-order) plus deadtime model and generate a dynamic model as a byproduct. Simulations presented illustrate the use of these algorithms in time-optimal control and the process models obtained. The algorithms are robust with respect to the measurement noise often encountered in real chemical processes and are inherently adaptive to changes in the process dynamics with time. An experimental application to the heatup of a laboratory fiber spinning apparatus is presented.

Since these time-optimal control algorithms are easy to apply and give rapid, smooth responses, they will be of value even when obtaining minimum-time response is not a critical issue. Such applications include startup of continuous and batch processes. Moreover, they can be used to determine first- or second-order process models (as appropriate) from open-loop step response data without actually implementing time-optimal control.

Introduction

The time-optimal control problem is basically to determine the control action that will drive a process output from an initial state to a specified final steady state in minimum time. According to Pontryagin's (1962) minimum principle, the resulting time-optimal control policy involves switching the forcing alternately from one extreme to the other during the transient period. To apply time-optimal control to a process, an exact mathematical model is required to bring the process output to rest exactly at its specified final steady state. Unfortunately, the process dynamics of many chemical processes are either difficult to obtain or are described by high-order nonlinear equations. Time-optimal control for nonlinear equations is not well developed, and each design tends to be unique to its process.

Usually, a satisfactory nth-order linear model may be obtained by linearizing the nonlinear equations about steady-state conditions. The theory of time-optimal control for an nth-order linear model is well developed and requires (for systems with real eigenvalues) at most (n-1) switchings of the control between the extreme values of the control input. To determine the switching times, however, a nonlinear multipoint boundary-value problem with unspecified final time must be solved. Thus, even if a rigorous high-order model were available, the computational requirements would be excessive for practical application. Fortunately, the process dynamics of many chemical processes can be approximated with second-order models with time delay. This is of interest because the model is simple and because theoretically only one switch between the extremes of the control is required. Many authors (Koppel and Latour, 1965; Mellichamp, 1970; Hsu et al., 1972) have demonstrated the time-optimal control of second-order systems with time delay.

A time-optimal set point change for a second-order system is accomplished by applying full forward forcing at time $t=0$, switching to full reverse forcing at time t_1 and returning to conventional regulator control at time t_2, as shown in Figure 1. The problem is to determine the switching times, t_1 and t_2, for a particular system. Traditionally, the application of time-optimal control techniques to second-order systems has been carried out in either of two ways. If the dynamic parameters of the system are known, t_1 and t_2 can be precalculated before the start of the set point change. The resulting open-loop con-

Correspondence concerning this article should be addressed to S. S. Melsheimer.

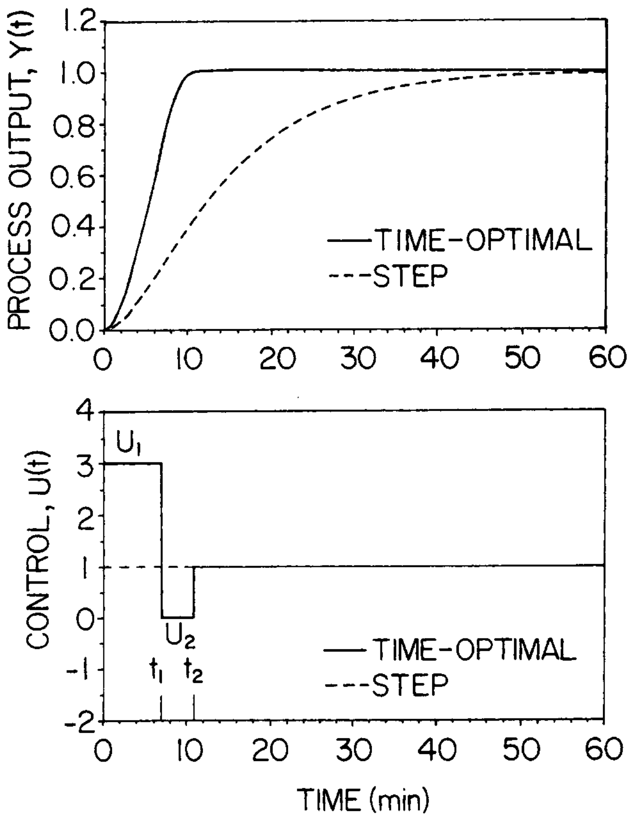

Figure 1. Time-optimal control of a second-order system.

trol law depends on the process elapsed time and the estimated model parameters. The controlled variable, which is the process output, is neither measured nor fed back to the controller. Thus, the performance of open-loop time-optimal control depends heavily on the accuracy of the model parameters. This is a major drawback of open-loop time-optimal control.

Alternatively, a feedback time-optimal control algorithm can be used. Here, a switching curve in the state plane [(dy/dt) vs. y] is computed using the estimated model parameters. The switching times are then determined by comparing the actual state of the plant, using measured state information, to the switching curve. With this approach, slight deviations of the state tend to be self-correcting. This form is preferable in practical implementation, although it still requires prior knowledge of the parameters of a dynamic process model.

It is highly desirable that a time-optimal controller be able to handle systems for which accurate dynamic models are not available. Real chemical process systems change with time and therefore a mathematical model that is accurate on one day may not be accurate the next day. Further, most chemical processes are described by nonlinear equations. Although linear system equations may be obtained by linearization around a steady-state operating condition or by plant test when the steady-state setpoint is changed the linear model may no longer accurately represent the system. Thus, model dependence implies heavy maintenance requirements. Several investigators (for example, Hsu et al., 1972) have developed methods of eliminating the requirement for prior knowledge of model dynamics. Most such methods rely on the fitting of model parameters with data obtained from a previous setpoint change or from the initial portions of the system response after a new set point change has begun.

Beard et al. (1974) devised a simple algorithm for time-optimal control of chemical processes which can be approximated by a second-order lag plus deadtime model. Knowledge of the unsteady-state model dynamic parameters, namely, ω_n (natural frequency) and ζ (damping ratio), is not required because the algorithm makes use of a dimensionless phase plane ($t\,dy/dt$ vs. y) on which switching curves are independent of system parameters for a given forcing. Using model deadtime as a tuning parameter, the algorithm was also shown to give good suboptimal control of nonlinear systems and systems of higher than second-order.

With no dynamic system model required, there is no need for extensive process testing and data fitting before the application of the Beard algorithm to a new process. Moreover, in systems with changing dynamic parameters (for example, fouling in a heat exchanger), the system does not have to be remodeled. The Beard algorithm also allows the coefficients of a second-order plus deadtime model to be computed from the switching times of a time-optimal setpoint change.

However, in applying the Beard time-optimal control algorithm (or other second-order feedback time-optimal control algorithms) to a real chemical process, only one output variable is usually measured, and a second state variable is obtained by differentiating the measured output signal. Noise and disturbances are invariably present in a real chemical process, and it is very difficult to estimate the first derivative of the measured output even if corrupted with only moderate levels of random noise. Such algorithms are particularly sensitive to fluctuations of (dy/dt) occurring late in time (near the first switch).

This work presents new algorithms which use a dimensionless phase plane formulation in terms of the integral of the process output rather than its derivative. This provides greatly improved tolerance of noise compared to the original Beard algorithm. These algorithms are readily implemented on small computers, are easy to apply and give rapid, smooth responses. They will be of value even when obtaining minimum time response is not a critical issue. Such applications include startup of continuous and batch processes. Moreover, they can be used to determine first- or second-order process models (as appropriate) as a byproduct of a setpoint change or from open-loop step response data obtained without actually implementing time-optimal control. Thus, they have utility as process identification tools as well as for time-optimal control.

Integral Form Algorithms

Two algorithms are developed below. The first, which is for the case of the general second-order plus deadtime system, does require an estimated deadtime. The second algorithm is for the case of a critically damped second-order plus deadtime system, and requires no *a priori* knowledge of the deadtime. The two algorithms can be used in combination (von Westerholt et al., 1989) to minimize the need for deadtime estimation.

A third algorithm has been developed which is applicable to first-order systems with deadtime. This algorithm, which may be used when the system dynamics are known to be well represented as first-order, is presented in Appendix B. For most chemical process applications, it is expected that the algorithms which permit second-order dynamics will be preferable.

Algorithm 1: second-order plus deadtime system

Most chemical engineering processes are characterized in the time domain by an S-shaped process reaction curve and can be described by the second-order plus deadtime transfer function:

$$\frac{Y(s)}{U(s)}=G(s)=\frac{e^{-\theta s}}{(s/\omega_n)^2+2(\zeta/\omega_n)s+1}=\frac{\lambda_1\lambda_2 e^{-\theta s}}{(s-\lambda_1)(s-\lambda_2)} \tag{1}$$

where

λ_1,λ_2 = system eigenvalues

ζ = damping coefficient $=-1/2(\lambda_1+\lambda_2)/(\lambda_1\lambda_2)^{1/2}=-[1+(\lambda_2/\lambda_1)]/[2(\lambda_2/\lambda_1)^{1/2}]$

ω_n = natural frequency $=(\lambda_1\lambda_2)^{1/2}=|\lambda_1|(\lambda_2/\lambda_1)^{1/2}$

θ = deadtime

The system output, Y, and input, U, are normalized by the steady-state output change and input change, respectively. Thus, the process steady-state gain is implicit in the definition of these variables and must be known *a priori*. It should be noted that while most chemical processes exhibit overdamped behavior, Eq. 1 and the subsequent development are also applicable to underdamped (complex eigenvalue) cases as well.

Since the deadtime has no effect on the switching times, it will be neglected in the initial development. The second-order system is then described by:

$$\frac{d^2y}{dt^2}-(\lambda_1+\lambda_2)\frac{dy}{dt}+\lambda_1\lambda_2 y=\lambda_1\lambda_2 u \tag{2}$$

with constraints

$$y(0)=0,\ \frac{dy(0)}{dt}=0,$$

$$y(t_2)=1,\ \frac{dy(t_2)}{dt}=0.$$

The time-optimal control of the system of Eq. 2 from the initial steady state to a new final state is well known (Pontryagin et al., 1962). If the eigenvalues are real, only one switch between the extremes of the manipulated variable is required. If the eigenvalues are complex this is not generally true, but Minnick (1984) showed that only one switch is required if the system is initially at rest. An equation for the time at which the control should be reversed, t_1, may be obtained by integrating Eq. 2 to time t_1 using $u=u_1$, then integrating it again from t_2 to t_1 using $u=u_2$. Applying the constraints specified with Eq. 2 leads to the implicit equation for t_1:

$$e^{(\lambda_2/\lambda_1)\lambda_1 t_1}+\frac{u_2}{u_1}-1+\frac{1-u_2}{u_1}\left\{\frac{u_1-u_2-u_1 e^{\lambda_1 t_1}}{1-u_2}\right\}^{(\lambda_2/\lambda_1)}=0 \tag{3}$$

where

u_1 = first control input

u_2 = second control input

Appendix A provides a more detailed derivation of Eqs. 3-5. The second forcing, u_2, is applied from time t_1 until time t_2, which is given by:

$$t_2=t_1-\ln\left[\frac{u_2+u_1(e^{\lambda_1 t_1}-1)}{(u_2-1)}\right]/\lambda_1 \tag{4}$$

At time t_2, the manipulated input is switched to the new steady-state value ($u=1$) or, more likely, control is transferred to a regulator algorithm.

Note in Eq. 3 that a dimensionless switching time ($\lambda_1 t_1$) is a function of only the eigenvalue ratio (λ_2/λ_1) for a specified set of forcings, u_1 and u_2. Then, the process output and its geometric average at $t=t_1$ are given by:

$$y(t_1)=u_1\left\{1+\frac{[\lambda_2 e^{\lambda_1 t_1}-\lambda_1 e^{\lambda_2 t_1}]}{\lambda_1-\lambda_2}\right\}$$

$$=u_1\left\{1+\frac{\left[\left(\frac{\lambda_2}{\lambda_1}\right)e^{\lambda_1 t_1}-e^{(\lambda_2\lambda_1)\lambda_1 t_1}\right]}{1-\left(\frac{\lambda_2}{\lambda_1}\right)}\right\} \tag{5}$$

and

$$Y_{\text{avg}}(t_1)=\frac{1}{t_1}\int_0^{t_1} y\,dt=u_1\left[1+\frac{1}{\lambda_1 t_1\left(1-\left(\frac{\lambda_2}{\lambda_1}\right)\right)}\times\left\{\frac{\lambda_2}{\lambda_1}[e^{\lambda_1 t_1}-1]-\frac{\lambda_1}{\lambda_2}[e^{(\lambda_2/\lambda_1)\lambda_1 t_1}-1]\right\}\right]. \tag{6}$$

For the specified u_1 and u_2, Eqs. 3, 5 and 6 can be used to compute a sequence of points $[y(t_1), Y_{\text{avg}}(t_1)]$ as a function of the eigenvalue ratio (λ_2/λ_1) which corresponds to the damping coefficient as shown below Eq. 1. This sequence of $[y(t_1), Y_{\text{avg}}(t_1)]$ values constitutes the generalized second-order switching curve for this set of input forcings. Figure 2 shows such a curve for $u_1=2$ and $u_2=-2$. Figure 2 spans the range of damping coefficients from highly underdamped ($\zeta=0.01$) to highly overdamped ($\zeta=5$). Two important properties of this switching curve should be reiterated:

1. A given point on the switching curve corresponds to a specific λ_2/λ_1 ratio, or equivalently, a specific damping coefficient ζ
2. At the same point on the switching curve, the product $\lambda_1 t_1$ (or equivalently, $\omega_n t_1$) also has a unique value.

When the measured system trajectory intersects the switching curve, the point of intersection determines the values (λ_1/λ_2) and ($\lambda_1 t_1$). Since the time of intersection, t_1, is known, the values of λ_1 and λ_2 can be immediately computed. In fact, since these values are determined without actually effecting the second input, u_2, this provides the basis for an identification method of determining second-order models from open-loop step response data (Lin, 1988).

Deadtime. Frequently, fitting a second-order model to a chemical process requires incorporation of deadtime into the model. This may result from a physical deadtime in the process or from additional lags due to the process being higher than second-order.

In either case, for a model with deadtime θ, the manipulated

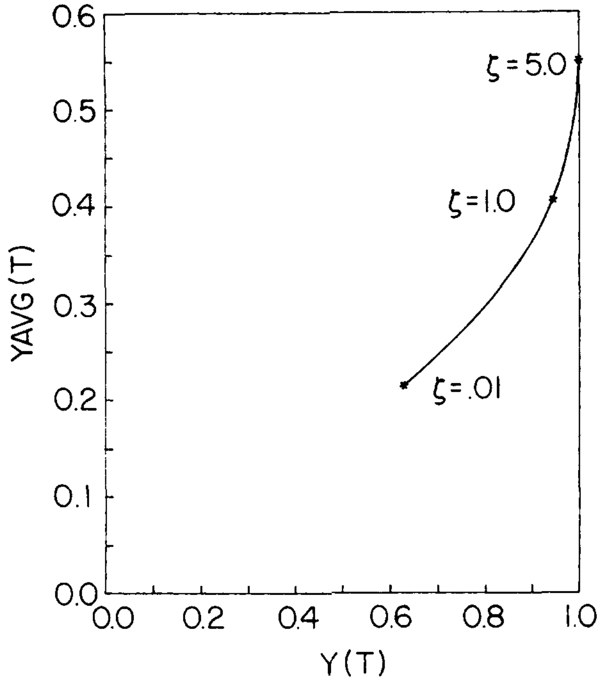

Figure 2. Dimensionless phase plane for the time-optimal control of second-order systems with $u_1=2$ and $u_2=-2$.

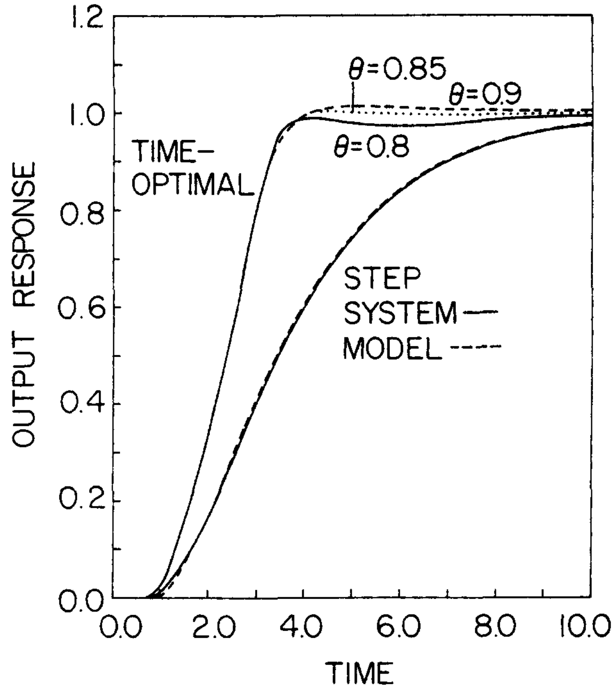

Figure 3. Tuning of algorithm I for the third-order system of Eq. 9.

variable should be switched θ units of time before the trajectory intersects the second-order switching curve. Since it is not possible to know the future output without knowing the dynamic parameters of the system, the following truncated Taylor Series estimates are used:

$$y(t+\theta)=y(t)+\theta\left.\frac{dy}{dt}\right|_t \tag{7}$$

$$Y_{avg}(t+\theta)=\frac{\left[\int_0^t y(t)dt+\theta\left[y(t)+0.5\theta\left.\frac{dy}{dt}\right|_t\right]\right]}{(t+\theta)}. \tag{8}$$

Thus, $[y(t+\theta),\ Y_{avg}(t+\theta)]$ is compared with the switching curve to determine the first switching time, t_1. Equation 4 then gives t_2.

It should be noted that the deadtime, θ, must be specified *a priori*. Methods have been devised for on-line estimation of the effective deadtime (Kuo, 1989). Alternatively, θ can be estimated off-line and then tuned by observing the controller performance. The apparent deadtime is adjusted until nearly equal overshoot and undershoot are obtained about the new steady state, $y=1$. The following examples illustrate this procedure.

Example 1. The system with transfer function:

$$G(s)=\frac{e^{-.5s}}{(0.5s+1)(s+1)(2s+1)} \tag{9}$$

is controlled by Algorithm I with $u_1=2$ and $u_2=-2$, and the responses are shown in Figure 3. An initial deadtime estimate of $\theta=0.8$ provided a satisfactory response, but tuning the apparent deadtime gave an improved value of 0.85. The resulting approximation model that was obtained is:

$$Gm(s)=\frac{e^{-0.85s}}{(1.265s+1)(1.857s+1)} \tag{10}$$

The step responses of the system and the final model are also shown in Figure 3 for comparison.

Example 2. Suppose that an undetected change occurs in the process so that its dynamic model becomes:

$$G(s)=\frac{e^{-0.5s}}{(0.5s+1)(1.2s+1)(2.5s+1)} \tag{11}$$

instead of Eq. 9. A setpoint change made without retuning the apparent model deadtime θ is shown in Figure 4. The approximation model now obtained is:

$$Gm(s)=\frac{e^{-0.85s}}{(1.863s+1)(1.882s+1)} \tag{12}$$

The system and model step responses are also shown in Figure 4. This demonstrates that the tuning of the algorithm is not very sensitive to the changes in the system dynamics.

Algorithm II: second-order critically damped plus deadtime system

If the process is assumed to be critically damped, it becomes feasible to directly determine the deadtime. The second-order plus deadtime critically damped model is given by:

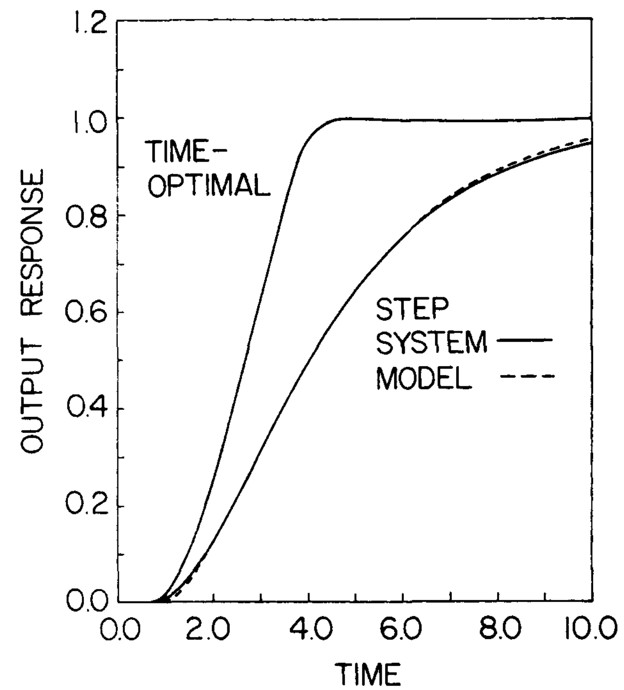

Figure 4. Response of the system of Eq. 11 controlled by algorithm I.

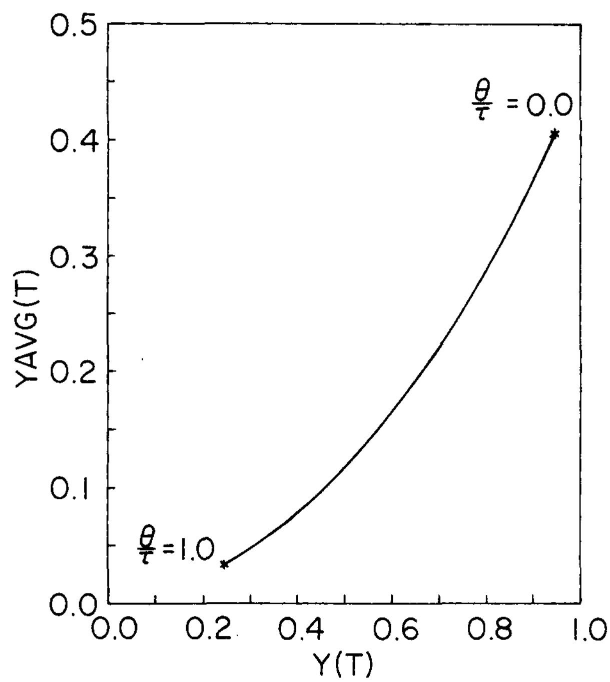

Figure 5. Dimensionless phase plane for time-optimal control algorithm II with $u_1=2$ and $u_2=-2$.

$$G(s)=\frac{Y(s)}{U(s)}=\frac{e^{-\theta s}}{(\tau s+1)^2} \tag{13}$$

with constraints

$$y(0)=0,\ \frac{dy}{dt}(0)=0,$$

$$y(t_2+\theta)=1,\ \frac{dy}{dt}(t_2+\theta)=0.$$

The time-optimal control of this system from $y(0)=0$ to $y(t_2+\theta)=1$ requires only one switch between the extremes of the manipulated variable (Pontryagin et al., 1962). If u_1 is the first control and u_2 is the second control in the time-optimal control sequence, the dimensionless time (t_1/τ) at which the control should be switched from u_1 and u_2 is given by the implicit equation (see Appendix A for a derivation of Eqs. 14 through 17):

$$\ln\left\{\frac{u_1-u_2-u_1e^{-t_1/\tau}}{1-u_2}\right\}+\frac{u_1(-t_1/\tau)e^{-t_1/\tau}}{u_1-u_2-u_1e^{-t_1/\tau}}=0. \tag{14}$$

Now, the second switching time, t_2, can be computed from:

$$t_2=t_1+\frac{t_1e^{-t_1/\tau}}{1-e^{-t_1/\tau}-\dfrac{u_2}{u_1}} \tag{15}$$

The process output and its geometric average at time t_1 (for $t_1>\theta$) are given by:

$$y(t_1)=u_1\left\{1-\left[1+\frac{(t_1-\theta)}{\tau}\right]e^{-(t_1-\theta)/\tau}\right\} \tag{16}$$

and

$$Y_{avg}(t_1)=\frac{\int_0^{t_1}y(t)dt}{t_1}$$
$$=u_1\left\{1-\frac{\tau}{t_1}\left[2+\frac{\theta}{\tau}-\left(2+\frac{t_1-\theta}{\tau}\right)e^{-(t_1-\theta)/\tau}\right]\right\}. \tag{17}$$

If u_1 and u_2 are specified, the dimensionless switching time (t_1/τ) is fixed by Eq. 14. Now, a switching curve with the coordinates $y(t_1)$ vs. $Y_{avg}(t_1)$, which depends upon (θ/τ) only, can be generated from Eqs. 14, 16 and 17. Figure 5 shows such a switching curve for $u_1=2$ and $u_2=-2$ over the range of (θ/τ) from 0.0 to 1.0. The model dynamic parameters τ and θ can be determined from the intersection of the system trajectory with the switching curve since:

1. A given point on the switching curve represents a specific ratio (θ/τ)
2. At the given point mentioned above, the ratio (t_1/τ) is fixed.

Tuning. Algorithm II is desirable for use with systems with deadtime, but may not adequately handle deviations from critically damped behavior. Thus, it is desirable to provide a tuning adjustment to increase its range of applicability. A tuning parameter α was introduced by altering the definition of $Y_{avg}(t)$ to $[\int y(t)dt/(t-\alpha)]$. The parameter α (which has no physical significance) can be adjusted so that the percentage overshoot is nearly equal to undershoot about the new steady state, $y=1$. This is demonstrated in the following example.

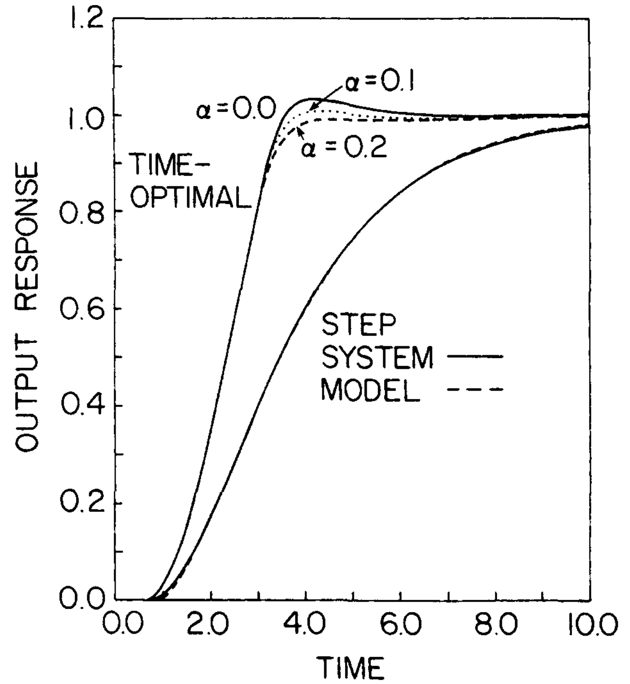

Figure 6. Tuning of algorithm II for the third-order system of Eq. 9.

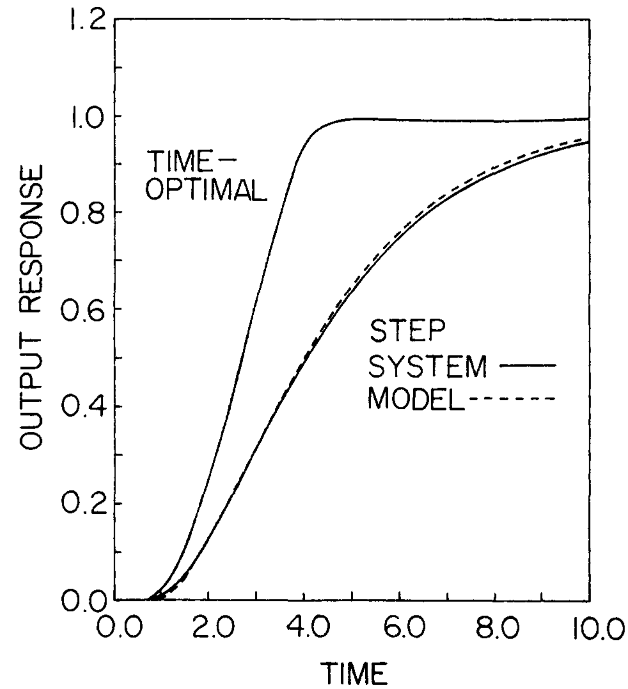

Figure 7. Response of the system of Eq. 11 controlled by algorithm II.

Example 3. Consider a system described by the transfer function of Eq. 9, with $u_1 = 2$ and $u_2 = -2$. The time-optimal control of the system of Eq. 9 from initial steady state to final steady state using Algorithm II is shown in Figure 6. It is seen that the adjustable parameter $\alpha = 0.2$ gives the best results. The model transfer function that was obtained is:

$$Gm(s) = \frac{e^{-0.835s}}{(1.583s+1)^2}. \tag{18}$$

The open-loop step responses of the system (Eq. 9) and the model (Eq. 18) are shown in Figure 6 for comparison.

Once the adjustable parameter α is determined, the time-optimal controller is rather insensitive to changes in process dynamics. This is because the algorithm itself compensates for the changes in process dynamics. This is demonstrated by the following example.

Example 4. Suppose that an undetected change occurred in the process dynamics so that the system description becomes Eq. 11. Applying the time-optimal controller without retuning gives the response shown in Figure 7. The model obtained from the time-optimal algorithm is now:

$$Gm(s) = \frac{e^{-0.866s}}{(1.891s+1)^2}. \tag{19}$$

It can be seen that the algorithm is not very sensitive to the system dynamic changes. The system (Eq. 11) and the model (Eq. 19) step responses are also shown in Figure 7.

Noisy processes

The measured outputs of chemical processes are often corrupted by measurement noise. To simulate a noisy process, a random noise $n(t)$ was produced digitally and was added to the output, as shown in Figure 8. The random function $n(t)$ was chosen such that it caused, at the measured output $y_m(t)$, a superimposed noise with peak-to-peak magnitude of 4% of the value of the setpoint change.

The value of $[(\int y_m(t)dt)/t]$ should approach the true value of $[(\int y(t)dt)/t]$ as the process elapsed time increases if the random noise has a zero mean. The random noise, however, will still affect $y_m(t)$, compromising its use as the other dimensionless phase plane coordinate. To avoid false intersection of the phase plane trajectory with the switching curve, a least-squares smoothing technique is used on $y_m(t)$. In the dimensionless phase plane, $y(t)$ vs. $[(\int y(t)dt)/t]$, the system trajectory is nearly linear over short intervals (see Figure 9, for example). Thus, $y(t)$ is estimated by:

$$y(t) = aY_{mavg}(t) + b \tag{20}$$

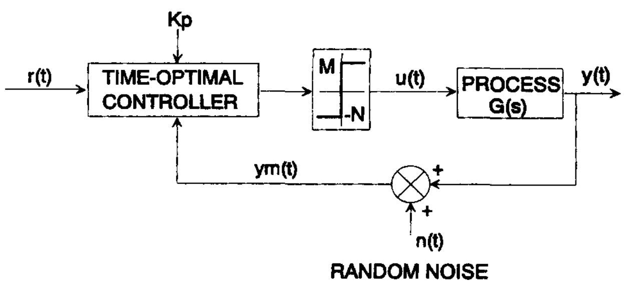

Figure 8. Simulation of a dynamic system with a random measurement noise.

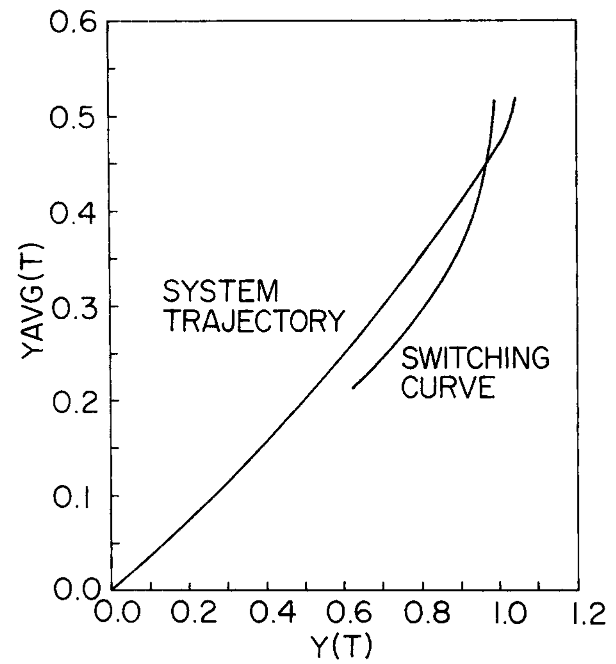

Figure 9. Dimensionless switching curve and trajectory for the system of Eq. 22 with 4% magnitude random noise controlled by algorithm I with $u_1=2$ and $u_2=-2$.

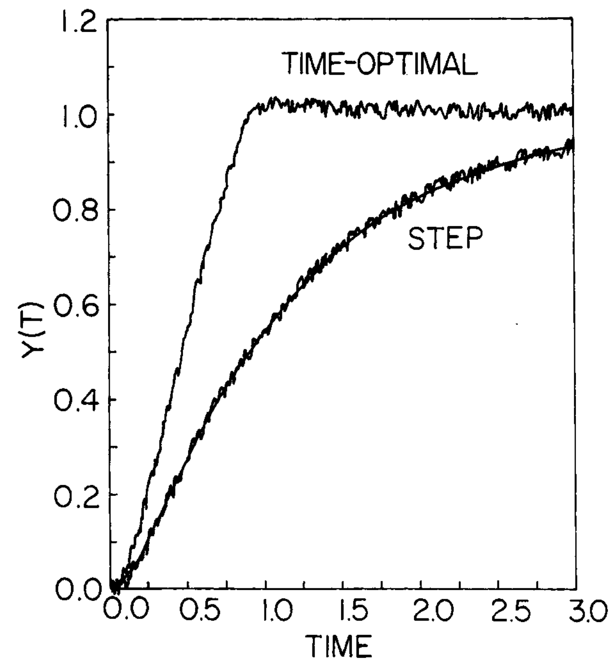

Figure 10. Responses of the system of Eq. 22 with a 4% magnitude random noise controlled by algorithm I.

where

$Y_{m\text{avg}}(t)$ = $[\int y_m(t)dt]/t$
$y(t)$ = estimated system output
$y_m(t)$ = measured system output

The coefficients a and b are determined by minimizing the sum of the squares of the deviations between the measured system response and the estimated system response:

$$E=\Sigma(y_{mi}-y_i)^2 \tag{21}$$

over the interval $i=j-m$ to $i=j$ where j is the current point. The sampling time and m are chosen in such a way that the assumption of the linearity between the $y(t)$ and $Y_{\text{avg}}(t)$ is good. While the smoothing technique shown in Eqs. 20-21 worked well, other schemes for smoothing $y_m(t)$ are certainly possible.

Example 5. A second-order overdamped system having the transfer function:

$$G(s)=\frac{1}{(s+1)(0.2s+1)} \tag{22}$$

with $u_1=2$ and $u_2=-2$ has its output signal corrupted by a random noise with a peak-to-peak magnitude of 4% of the magnitude of the setpoint change. Figure 9 shows the generalized switching curve for Algorithm I and the estimated system trajectory with sampling time equal to 0.02 and $m=30$. The resulting system time-optimal response is shown in Figure 10. A comparison of the system (with noise) and the model (smooth solid line) step responses is also shown in Figure 10.

Experimental Results

A temperature control system for a laboratory-scale batch melt spinning process was used to test Algorithm I. Figure 11 shows a schematic diagram of the melt spinning process. A metal conduction heating collar was placed around the cartridge containing the material to be spun and was heated by means of electrical heaters located inside the collar. To minimize heat losses to the atmosphere, insulation was placed around the heating collar and the cartridge. The material being spun was heat-sensitive, so the control objective was to drive the cartridge or melt temperature from room temperature to the operating temperature as quickly as possible without overshoot.

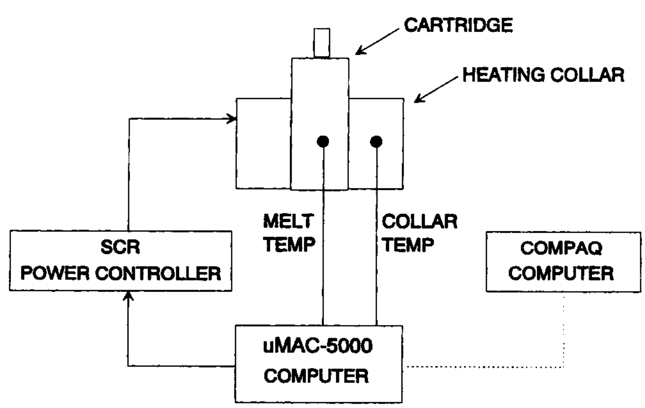

Figure 11. Temperature control and data acquisition system of a laboratory batch melt spinning process.

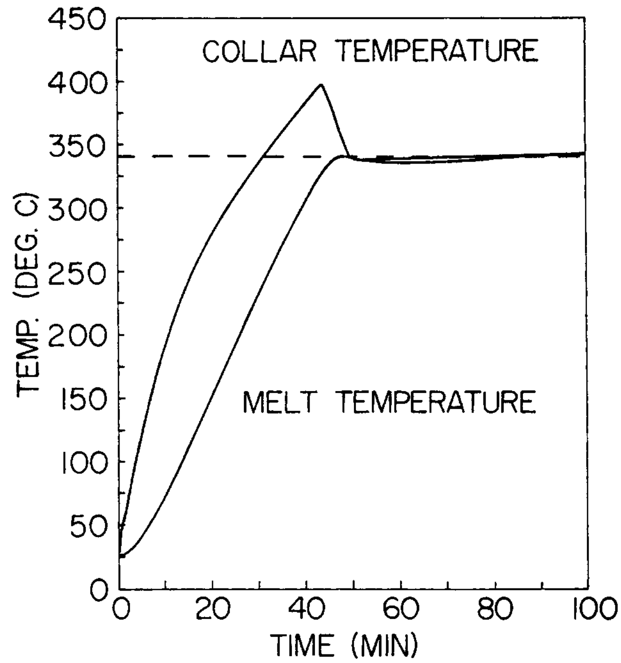

Figure 12. Responses of the cartridge temperature and heating collar temperature controlled by algorithm I with $u_1 = 2.45$ and $u_2 = 0$.

The cartridge and heating collar temperatures were measured by type-J thermocouples using an Analog Devices μMAC-5000 data acquisition system. The μMAC-5000 output a 0–20 ma control signal to a DAPC-1s SCR power controller which modulated the power supply to the heaters. The operating conditions and the data required by the model-independent time-optimal algorithm were:

Y_i = 26°C, initial cartridge temperature
Y_F = 340°C, desired cartridge temperature
K_p = 64°C/ma, steady-state gain
U_{max} = 12 ma, 60% of full range
U_{min} = 0 ma, 0% of full range
U_i = 0 ma, initial input
U_F = 4.9 ma $=(Y_F - Y_i)/K_p + U_i$, steady-state input

In terms of dimensionless variables:

$$y = \frac{Y - Y_i}{Y_F - Y_i}$$

$$u_1 = \frac{U_{max} - U_i}{U_F - U_i} = \frac{(U_{max} - U_i)K_p}{(Y_F - Y_i)} = 2.45$$

$$u_2 = \frac{U_{min} - U_i}{U_F - U_i} = 0$$

Using Algorithm I, the maximum forcing U_{max} was applied at time zero, the forcing was switched to U_{min} when the measured system trajectory intersected the dimensionless switching curve, and the new steady-state control, U_F, was applied at time t_2. The responses of the cartridge temperature and the heating collar temperature are shown in Figure 12. It may be noted that the cartridge (polymer) temperature comes quickly to the new steady state with virtually no overshoot, but the collar temperature does overshoot. The collar temperature responds almost immediately to changes in heater power, thus sharp changes are seen at about 42 min and 44.5 min, t_1 and t_2, respectively. The maximum forcing was limited to 60% of the physically available forcing in order to limit the collar temperature, since the surface of the melt material is in contact with the collar.

Conclusions

New time-optimal control algorithms have been developed which do not require *a priori* knowledge of the dynamic parameters and which are applicable to systems that can be described as second-order plus deadtime. Thus, the algorithms are inherently adaptive to variations in dynamic response resulting from process nonlinearities or changes in the process with time. Moreover, the parameters of a second-order model are readily derived from the results of a setpoint change and can be used, for example, in tuning a regulator controller. The algorithms are relatively robust with respect to measurement noise and provide for tuning adjustments to minimize the performance penalties that may arise when the process is more complex than second-order. Application to a laboratory fiber spinning apparatus has demonstrated the utility and robustness of new algorithms. Further study, however, is required to establish the capability of the algorithms to handle severe nonlinearities or to cope with significant external disturbances.

Notation

E = sum of squares of deviations
$G(s)$ = transfer function of system
$G_m(s)$ = transfer function of model
K_p = process gain
M = maximum (positive) value of manipulated variable
$n(t)$ = random noise signal
N = minimum (negative) value of manipulated variable
$r(t)$ = setpoint
s = Laplace operator
t_1 = time of the first switch of manipulated variable
t_2 = time of the second switch of manipulated variable
$u(t)$ = manipulated variable (normalized)
u_1 = first control used in time-optimal control sequence (normalized)
u_2 = second control used in time-optimal control sequence (normalized)
U_F = control input value at final steady state
U_i = control input value at initial steady state
U_{max} = maximum control input value
U_{min} = minimum control input value
$y(t)$ = system output (normalized)
y_i = sampled value of $y(t)$
$y_m(t)$ = measured system output (normalized)
y_{mi} = sampled value of $ym(t)$
$Y_{avg}(t)$ = $[\int y(t)dt]/t$
$Y_{mavg}(t)$ = $[\int ym(t)dt]/t$
Y_F = final system output (not normalized)
Y_i = initial system output (not normalized)

Greek letters

α = tuning parameter for Algorithm II
ζ = damping ratio
θ = deadtime
$\lambda, \lambda_1, \lambda_2$ = model eigenvalues
τ = model dominant time constant
ω_n = natural frequency

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Appendix A: Derivation of Equations for Switching Curves

The time-optimal policy for linear systems with real eigenvalues is well known. Specifically, for an nth-order system, at most (n-1) switches are required between the extremes of the manipulated variable followed by a switch to the final steady-state value of the input (Pontryagin et al., 1962; Athens and Falb, 1966). Thus, for second-order systems, only one switch is required. While this is not true in general for systems with complex eigenvalues, Minnick (1984) showed that, for systems which are initially at rest, the optimal policy does have only a single switch. Regardless of the nature of the eigenvalues, the model-independent algorithms require the system to be initially at rest, and consequently the results are applicable to systems with complex as well as real eigenvalues.

Thus, the problem is to solve for the first switching time, t_1, and to compute the time, t_2, at which the new steady state is attained.

Distinct eigenvalues (algorithm I)

Consider the linear second-order system:

$$\frac{d^2y}{dt^2}-(\lambda_1+\lambda_2)\frac{dy}{dt}+\lambda_1\lambda_2 y=\lambda_1\lambda_2 u \tag{A1}$$

with the constraints:

$$y(0)=0,\ dy/dt(0)=0,$$
$$y(t_2)=1,\ dy/dt(t_2)=0,$$
$$-N\leq u\leq M,\ M>0,\ N\geq 0.$$

and

$$\lambda_1\neq\lambda_2\neq 0.$$

Now, let $z_1=y$ and $z_2=dy/dt$. Then, Eq. A1 becomes Eq. A2:

$$\dot{\bar{z}}=\begin{bmatrix}0 & 1\\ -\lambda_1\lambda_2 & \lambda_1+\lambda_2\end{bmatrix}\bar{z}+\begin{bmatrix}0\\ \lambda_1\lambda_2\end{bmatrix}u \tag{A2}$$

Transforming Eq. A2 to new decoupled state variables x_1 and x_2 using:

$$\bar{x}=\begin{bmatrix}-1 & 1/\lambda_2\\ -1 & 1/\lambda_1\end{bmatrix}\bar{z}+\begin{bmatrix}1\\ 1\end{bmatrix} \tag{A3}$$

gives:

$$\dot{\bar{x}}=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}\bar{x}+\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix}u-\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix}, \tag{A4}$$

with constraints:

$$x_1(0)=x_2(0)=1$$

and

$$x_1(t_2)=x_2(t_2)=0.$$

Integrating Eq. A4 from $t=0$ to $t=t_1$ using $u=u^1$ gives:

$$x_1^s=u_1(e^{\lambda_1 t_1}-1)+1$$

and

$$x_2^s=u_1(e^{\lambda_2 t_1}-1)+1 \tag{A5}$$

where
x_1^s, x_2^s = state values at time t_1
u_1 = control value from $t=0$ to $t=t_1$

Again integrating Eq. A4, this time backwards from $t=t_2$ to $t=t_1$ gives two other equations for x_1^s, x_2^s:

$$x_1^s=(u_2-1)[e^{\lambda_1(t_1-t_2)}-1]$$

and

$$x_2^s=(u_2-1)[e^{\lambda_2(t_1-t_2)}-1] \tag{A6}$$

where

$$u_2=\text{control value from } t=t_2 \text{ to } t=t_1.$$

Combining Eqs. A5 and A6 to eliminate x_1^s, x_2^s and t_2 yields:

$$e^{\lambda_2 t_1}+u_2/u_1-1+\frac{1-u_2}{u_1}\left\{\frac{[u_1-u_2-u_1e^{\lambda_1 t_1}]}{1-u_2}\right\}^{\lambda_2/\lambda_1}=0, \tag{A7}$$

which is Eq. 3. Equation 4 is obtained by equating x_{1s} in Eqs. A5 and A6, and solving for t_2. Substituting Eq. A5 into Eq. A3 and solving for the output ($y=z_1$) give Eq. 5. Finally, integrating $y(t)$ from $t=0$ to $t=t_1$ and dividing by t_1, give Eq. 6.

Critically damped with deadtime (algorithm II)

Consider the linear second-order critically damped system:

$$\tau^2\frac{d^2y(t)}{dt^2}+2\tau\frac{dy(t)}{dt}+y(t)=u(t) \tag{A8}$$

with constraints

$$y(0)=0;\ dy(0)/dt=0,$$
$$y(t_2)=1;\ dy(t_2)/dt=0,$$
$$-N\leq u\leq M;\ M>0 \text{ and } N\geq 0.$$

Of course, the model could be just as well expressed in terms of the eigenvalue $\lambda=-(1/\tau)$. Let

$$y_1=dy(t)/dt;\ y_1(0)=0;\ y_1(t_2)=0.$$

The solution of $y(t)$ from $t=0$ to $t=t_1$ and its first derivative are given by:

$$y(t_1)=u_1[1-(1+t_1/\tau)e^{-t_1/\tau}], \tag{A9}$$

$$y_1(t_1)=u_1/\tau(t_1/\tau)e^{-t_1/\tau} \tag{A10}$$

where
t_1 = the first switching time
u_1 = control value used from $t=0$ to $t=t_1$

The solution of $y(t)$ from $t=t_1$ to $t=t_2$, and its first derivative, are:

$$y(t_2)=1=(y(t_1)-u_2)G_1+\left[y_1(t_1)+\left(\frac{y(t_1)-u_2}{\tau}\right)\right](t_2-t_1)G_1+u_2, \tag{A11}$$

$$y_1(t_2)=0=\tau y_1(t_1)-(t_2-t_1)\left[y_1(t_1)+\left(\frac{y(t_1)-u_2}{\tau}\right)\right] \tag{A12}$$

where
G_1 = $e^{-(t_2-t_1)/\tau}$,
t_2 = final switching time
u_2 = control value from $t=t_1$ to $t=t_2$

Combining Eqs. A9, A10, A11 and A12 so as to eliminate $y(t_1)$, $y_1(t_1)$ and t_2 yields:

$$\ln\left\{\frac{u_1-u_2-u_1e^{-t_1/\tau}}{1-u_2}\right\}+\frac{u_1(-t_1/\tau)e^{-t_1/\tau}}{u_1-u_2-u_1e^{-t_1/\tau}}=0 \tag{A13}$$

which is Eq. 14. For given values of u_1 and u_2, Eq. 13 gives a single dimensionless switching time, (t_1/τ). Equations A9 and A10 can be substituted into Eq. A12 to obtain Eq. 15 for the second switching time, t_2.

Now, if the system has a pure time delay θ, the system output at time t_1 is given by:

$$y(t_1)=u_1\left[1-\left(1+\frac{(t_1-\theta)}{\tau}\right)e^{-(t_1-\theta)/\tau}\right], \tag{A14}$$

which is Eq. 16. Integrating $y(t)$ from $t=0$ to $t=t_1$ and dividing by t_1 give Eq. 17. Since (t_1/τ) is uniquely determined by Eq. A13 for a given set of forcings (u_1,u_2), a switching curve can be created as a function solely of (θ/τ) on the modified phase plane.

Appendix B: Time-Optimal Control Algorithm for First-Order Plus Deadtime Systems

Time-optimal control of systems that are first-order is trivial; full forcing is applied until the desired output (new setpoint) is reached, and then the control is switched to the new steady-state value. If the system includes deadtime, however, the switch must be made prior to the point at which the measured output reaches the new setpoint. An algorithm similar to algorithm II for critically damped plus deadtime systems has been developed for this case.

Given the linear first-order system:

$$\tau\frac{dy(t)}{dt}+y(t)=u(t) \tag{B1}$$

with constraints:

$$y(0)=0;\ dy(0)/dt=0,$$
$$y(t1)=1;\ dy(t_1)/dt=0,$$
$$-N\leq u\leq M;\ M>0 \text{ and } N\geq 0.$$

The time-optimal solution as noted above is to apply the forcing $u=M$ until time t_1 at which $y=1$. The solution of Eq. B1 is thus:

$$y(t_1)=u_1(1-e^{-t_1/\tau}). \tag{B2}$$

Therefore,

$$\left(\frac{t_1}{\tau}\right)=\ln[u_1/(u_1-1)] \tag{B3}$$

For a system with deadtime θ the process output at time t_1 is given by:

$$y(t_1)=u_1(1-e^{-(t_1-\theta)/\tau})=u_1(1-e^{\theta/\tau}e^{-t_1/\tau}). \tag{B4}$$

assuming $t_1>\theta$.

Integrating Eq. B4 from $t=0$ to $t=t_1$ and dividing by t_1:

$$Y_{\text{avg}}(t_1)=\frac{\int_0^{t_1} y\,dt}{t_1}=u_1\left\{1-\frac{\theta}{t_1}-\frac{\tau}{t_1}\left[1-e^{-(t_1-\theta)/\tau}\right]\right\}$$
$$=u_1\left\{1-\frac{\tau}{t_1}\left[1+\frac{\theta}{\tau}-e^{-t_1/\tau}e^{\theta/\tau}\right]\right\}. \tag{B5}$$

For a specified u_1, Eq. B3 gives the dimensionless switching time (t_1/τ). Equations B4 and B5 then define a switching curve on the (y,Y_{avg}) phase plane, which is a function only of the parameter (θ/τ). Thus, the trajectory of a first-order plus deadtime system is traced on this phase plane until it intersects the switching curve. At this time, the switch is made to the new steady-state input. Equation B3 then determines τ. Equation B4 can then be solved for the deadtime, θ:

$$\theta=\tau\ \ln[u_1-y(t_1)]/(u_1-1). \tag{B6}$$

Note that for this algorithm (or any feedback algorithm) to be applied, the switching time, t_1, must exceed the deadtime, θ.

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